

Solutions to Maths workbook – 1 | Trigonometric Identities and Equations

Level - 2	Daily Tutorial Sheet - 8
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101.(A) $5^x + 5^{-x} \geq 2 \Rightarrow \sin(e^x) \geq 2$

which is not possible for any real value of 'x'. Thus the given equation has no solution.

102.(D) Consider: 1st equation i.e., $2 \sin^2 x + 3 \sin x - 2 > 0$

$$\Rightarrow (2 \sin x - 1)(\sin x + 2) > 0 \Rightarrow (2 \sin x - 1) > 0 \text{ [As } \sin x + 2 > 0 \text{ } x \in R]$$

$$\Rightarrow \sin x > \frac{1}{2} \Rightarrow x \in \left(\frac{\pi}{6}, \frac{5\pi}{6}\right) \dots(i)$$

Consider: $x^2 - x - 2 < 0 \Rightarrow (x - 2)(x + 1) < 0 \Rightarrow -1 < x < 2 \dots(ii)$

From (i) and (ii), Now As, $2 < \frac{5\pi}{6}$, we obtain that 'x' must lie in $\left(\frac{\pi}{6}, 2\right)$.

103.(A) Consider : $y = 5x^2 + 2x + 3 = 5\left[x^2 + \frac{2}{5}x + \frac{3}{5}\right] = 5\left[\left(x + \frac{1}{5}\right)^2 + \frac{3}{5} - \frac{1}{25}\right] = 5\left(x + \frac{1}{5}\right)^2 + \frac{14}{5} > 2$

104.(D) We have $a_1 + a_2 \cos 2x + a_3 \sin^2 x = 0 \Rightarrow a_1 + a_2 \cos 2x + a_3 (1 - \cos 2x)/2 = 0$

Which is zero for all values of 'x' : If $a_1 = -\frac{a_3}{2} = -a_2$ or $a_1 = \frac{-k}{2}$, $a_2 = \frac{k}{2}$, $a_3 = k$

For any $k \in R$. Here the required no. of triplets is infinite.

105.(C) We have, $\sin^4 x - (k + 2) \sin^2 x - (k + 3) = 0$

$$\Rightarrow \sin^2 x = \frac{(k + 2) \pm \sqrt{(k + 2)^2 + 4(k + 3)}}{2} = \frac{(k + 2) \pm (k + 4)}{2}$$

$$\Rightarrow \text{Either } \sin^2 x = k + 3 \quad \text{or} \quad \sin^2 x = -1$$

$$\Rightarrow 0 \leq \sin^2 x \leq 1 \quad \text{or} \quad \text{not possible}$$

$$\Rightarrow 0 \leq k + 3 \leq 1 \quad \Rightarrow \quad -3 \leq k \leq -2$$

106.(AC) We have, $\sin^{50} x - \cos^{50} x = 1 \Rightarrow \sin^{50} x = 1 + \cos^{50} x$

Now as, $\sin^{50} x \leq 1$ and $1 + \cos^{50} x \geq 1$,

Therefore two sides are equal only if, $\sin^{50} x = 1 = 1 + \cos^{50} x$ i.e. $\sin^{50} x = 1$ and $\cos^{50} x = 0$

$$\Rightarrow x = n\pi + \frac{\pi}{2}, n \in I$$

107.(D) As, $\sin \theta - \cos \theta \neq 0$

$$\Rightarrow \tan \theta \neq 1 \Rightarrow \theta \neq \frac{\pi}{4}, \frac{5\pi}{4}, \text{ Also } \theta \notin \left\{0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi\right\} \text{ for } \tan \theta \text{ and } \cot \theta \text{ to be well defined.}$$

Now $\frac{(\sin \theta - \cos \theta)(\sin^2 \theta + \cos^2 \theta + \sin \theta \cdot \cos \theta)}{(\sin \theta - \cos \theta)} - \frac{\cos \theta}{\sqrt{\csc^2 \theta}} - 2 \tan \theta \cdot \cot \theta = -1$

$$\Rightarrow (1 + \sin \theta \cdot \cos \theta) - |\sin \theta| \cos \theta - 2 \tan \theta \cdot \cot \theta = -1$$

$$\Rightarrow 1 + \cos \theta (\sin \theta - |\sin \theta|) - 2 = -1 \Rightarrow \cos \theta (\sin \theta - |\sin \theta|) = 0 \Rightarrow \theta \in (0, \pi) - \left\{\frac{\pi}{4}, \frac{\pi}{2}\right\}$$

108.(B) $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2} \right] \Rightarrow -1 \leq \sin \theta \leq 1$ But here $0 < \sin \theta < 1$ [As $\log_a x$ is defined for $a > 0$ or $0 < a < 1$]

Now, $\log_{\sin \theta} \cos 2\theta = 2$

$\Rightarrow \cos 2\theta = \sin^2 \theta \Rightarrow 1 - 2 \sin^2 \theta = \sin^2 \theta \Rightarrow 3 \sin^2 \theta = 1$

$\Rightarrow \sin^2 \theta = \frac{1}{3} \Rightarrow \sin \theta = \frac{1}{\sqrt{3}}$ (as $0 < \sin \theta < 1$ so $\sin \theta \neq \frac{-1}{\sqrt{3}}$)

$\Rightarrow$ The given equation has a unique solution.

109.(AB) Let $\frac{\tan x}{1} = \frac{\tan y}{2} = \frac{\tan z}{3}; (\neq 0) = \lambda$ (say)

$\Rightarrow \tan x = \lambda, \tan y = 2\lambda, \tan z = 3\lambda$

$x + y + z = \pi \Rightarrow \tan x + \tan y + \tan z = \tan x \cdot \tan y \cdot \tan z$

$\Rightarrow 6\lambda = 6\lambda^3 \Rightarrow 6\lambda(\lambda^2 - 1) = 0 \Rightarrow \lambda = \pm 1$ ($\lambda \neq 0$)

$\Rightarrow \tan x = \pm 1, \tan y = \pm 2, \tan z = \pm 3 \Rightarrow \tan x + \tan y + \tan z = \pm 6$

$\Rightarrow$ Maximum value is 6 and minimum value is -6.

110.(D) $F(x) = \sin x + \cos x$... (i)

$$F\left(\frac{\pi}{10}\right) = \left[\sin\left(\frac{\pi}{10}\right) + \cos\left(\frac{\pi}{10}\right) \right] = [\sin 18^\circ + \cos 18^\circ]$$

$$= \left[\sqrt{2} \left(\frac{\sin 18^\circ}{\sqrt{2}} + \frac{\cos 18^\circ}{\sqrt{2}} \right) \right] = [\sqrt{2} \sin (45^\circ + 18^\circ)] = [\sqrt{2} \sin 63^\circ]$$

As $\sin 63^\circ > \sin 45^\circ \Rightarrow \sqrt{2} \sin 63^\circ > 1$ (ii) Now $-\sqrt{2} \leq \sqrt{2} \sin 63^\circ \leq \sqrt{2}$

From (ii) $1 < \sqrt{2} \sin 63^\circ \leq \sqrt{2} \Rightarrow [\sqrt{2} \sin 63^\circ] = 1 \Rightarrow F(x) = 1$

From (i)

$\Rightarrow \sin x + \cos x = 1 \Rightarrow \cos\left(x - \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} \Rightarrow x - \frac{\pi}{4} = 2n\pi \pm \frac{\pi}{4} \Rightarrow x = 2n\pi, 2n\pi + \frac{\pi}{2}, n \in I$